TRINITY COLLEGE

Mathematics Methods Unit 3/4
Test 6 2022

Section 1 Calculator Allowed

Sampling

STUDENT'S NAME Solutions

DATE: Monday 5th September **TIME:** 50 minutes **MARKS**: 51

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

To study the effectiveness of yoga for reducing stress levels, a researcher measured the stress levels of 50 people who had just enrolled in a 10-week introductory yoga course, and then measured their stress levels at the end of the course.

(a) Do you think that this sample will be representative of the general population? Explain your answer. [2]

No
This sample would be biased as choosing people from a yoga course.

✓ No

✓ Mention of bias

(b) How would you suggest that the sample could be chosen? [1]

Random sample

✓ Any valid random sample method.

2. (4 marks)

A sample of n people were asked whether they thought that income tax in Australia was too high, and 90% said yes.

(a) What is the value of the sample proportion $\hat{p}$? [1]

$\hat{p} = 0.9$ ✓ $\hat{p}$

(b) Write down an expression for E, the margin of error for this estimate at the 95% confidence interval, in terms of n. [1]

$E_1 = 1.960\sqrt{\frac{0.9(0.1)}{n}}$ ✓ E

(c) If the number of people in the sample were doubled, what would be the effect on the margin of error E? [2]

$E_2 = 1.960\sqrt{\frac{0.9(0.1)}{2n}}$

$= 1.960\sqrt{\frac{1}{2}}\sqrt{\frac{0.9(0.1)}{n}}$

$\therefore E_2 = \frac{1}{\sqrt{2}} E_1$ ✓ New E ✓ $\times \frac{1}{\sqrt{2}}$

3. (6 marks)

A random sample of 100 people indicated that 19% had taken a plane flight in the last year.

(a) Determine a 90% confidence interval for the proportion of the population that had taken a plane flight in the last year. [2]

$(0.1255, 0.2545)$ ✓ lower ✓ upper

Assume the 19% sample proportion applies to the whole population.

(b) A new sample of 200 people was taken and X = the number of people who had taken a plane flight in the last year was recorded. Give a range using the 90% confidence interval, within which you would expect X to lie. [1]

$(0.1255 \times 200, \ 0.2545 \times 200)$

$[25.1, \ 50.9]$ ✓ range of X values

(c) Determine the probability that in a random sample of 120 people, the number who had taken a plane flight in the last year was greater than 26. [3]

$X \sim B(120, 0.19)$ ✓ stating binomial

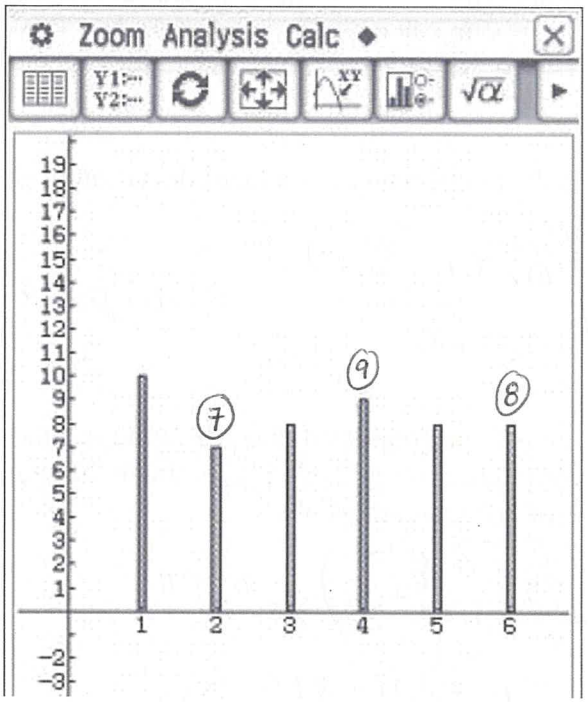
$P(X > 26) = P(X \geq 27)$ since discrete ✓ correct $P(X \geq 27)$

$= 0.1928$ ✓ probability

4. (4 marks)

The graph on the calculator screen shot below shows the results of a simulation of the tossing of a standard six-sided die, 50 times.

Simulated results of 50 tosses of a standard six-sided die



(a) Describe the type of probability distribution related to this simulation above. [1]

Uniform distribution

✓ Uniform

(b) Calculate the proportion of even numbers recorded in this simulation above. [1]

7 + 9 + 8 = 24

$p = {}^{24}/_{50}$

✓ p

(c) This simulation in part (a) is repeated another 500 times and the proportion (p) of even numbers is recorded for each simulation. Comment on the key features of a typical graph showing the results of 500 simulations. [2]

Binomial Distribution (Even or Not Even)

Distribution tends towards a Normal distribution as number of simulations increases with $p = 0.5$.

✓ Normal

✓ parameter

5. (10 marks)

A random survey was conducted to estimate the proportion of mobile phone users who favoured smart phones over standard phones. It was found that 283 out of 412 people surveyed preferred a smart phone.

(a) Determine the sample proportion $\hat{p}$ of those in the survey who preferred a smart phone. [1]

$\hat{p} = \frac{283}{412}$ ✓ $\hat{p}$

(b) Use the survey results to estimate the standard deviation of $\hat{p}$. [2]

$\sigma = \sqrt{\frac{\frac{283}{412}\left(1 - \frac{283}{412}\right)}{412}} \approx 0.0228$

✓ σ rule

✓ correct σ

(c) A follow-up survey is to be conducted to confirm the results of the initial survey. Working with a confidence interval of 95%, estimate the sample size necessary to ensure the margin of error is at most 4%. [3]

$1.96\left(\sqrt{\frac{\frac{283}{412}\left(1 - \frac{283}{412}\right)}{n}}\right) = 0.04$

✓ correct z

$n = 516.785$

✓ solving for n

$\therefore n = 517$

✓ round n correctly

The 90% confidence interval of the sample proportion $\hat{p}$ from the initial survey is $0.649 \leq \hat{p} \leq 0.725$.

(d) For each of the following samples, calculate the 90% confidence interval, then compare these to the 90% confidence interval of the initial sample. Using your results, state whether they suggest that the samples came from the same population or not.

(i) A random sample of 365 people at a shopping centre found that 258 had a preference for a smart phone. [2]

$\hat{p} = \frac{258}{365} = 0.71$

$(0.668, 0.746)$ ✓ CI

CI overlaps ∴ sampling from same population. ✓ comment

(ii) A random sample of 78 people at a retirement village found that 32 had a preference for a smart phone. [2]

$\hat{p} = \frac{32}{78} = 0.41$ $(0.319, 0.5019)$

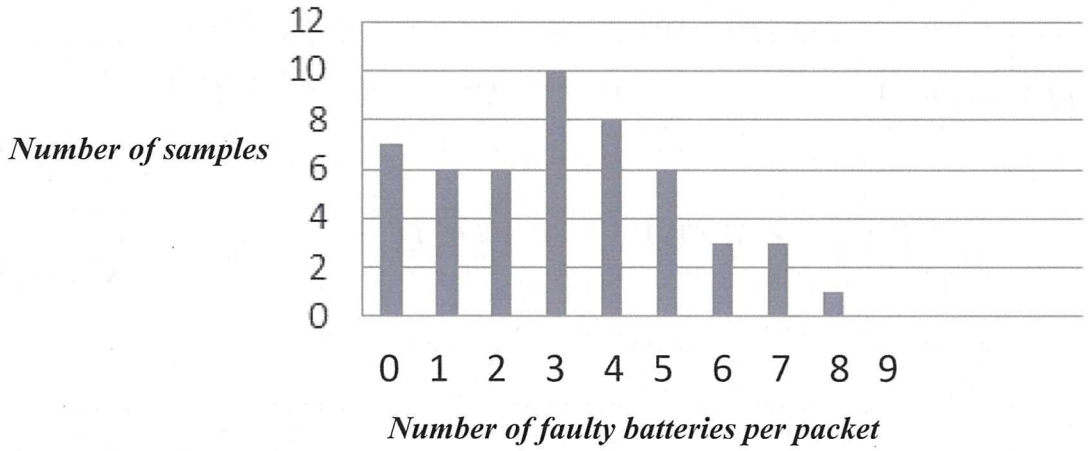
✓ CI

✓ comment

$\hat{p}$ & CI quite different ∴ biased survey from inside retirement village

6. (6 marks)

The graph below shows the number of faulty batteries per packet of 50 AAA batteries, when 50 packets are sampled at random.



(a) Identify the type of distribution of X = the number of faulty batteries per packet of 50 AAA batteries. [1]

Binomial (accept Bernoulli) ✓

A manufacturer of AAA batteries assumes that 99% of the batteries produced are fault-free.

Ten samples of 50 packets of 50 AAA batteries are selected at random and tested. The number of faulty batteries in each of the 10 random samples is shown below.

Sample	1	2	3	4	5	6	7	8	9	10
Number of faulty batteries	34	28	22	38	28	30	22	16	28	30

$50 \times 50 = 2500$ Σ faulty $= 276$ 1% of 2500 $= 25$

(b) Using the assumption that 99% of batteries are fault free calculate the 95% confidence interval for the proportion of faulty batteries expected when sampling. [3]

$p = \frac{25}{2500} = 0.01$ ✓ correct p

(0.0060997, 0.0139003) ✓ lower ✓ upper

(c) Decide which of the samples, if any, lie outside the 95% confidence level. [2]

Sample 4

$\frac{38}{2500} = 0.0152$ ✓ sample 4

outside of CI ✓ reason

(15.24, 34.75)

7. (6 marks)

According to recent research, 14% of Australians are left-handed.

(a) If a large number of random samples of 35 Australians are collected, what proportion of these samples are expected to contain less than 10% of left-handers? [2]

$\hat{p} \sim N(p, s^2)$ $\quad p = 0.14 \quad s = 0.0587$

✓ Normal & parameters

$P(\hat{p} < 0.1) = 0.2478$

✓ correct P

(b) If a large number of random samples of 300 Australians are collected, what proportion of these samples are expected to contain between 12% and 15% of left-handers? [2]

$\hat{p} \sim N(p, s^2)$ $\quad p = 0.14 \quad s = 0.0200$

✓ new s

$P(0.12 < \hat{p} < 0.15) = 0.5328$ ✓ correct P

a b

(c) One of the answers to ~~(b)~~ or ~~(c)~~ should be treated with some caution. State which answer and explain why. [2]

(a) should be treated with caution

✓ (a)

n is only 35

$np < 10$

✓ reason

so assumed normal distribution of $\hat{p}$ is unlikely to be suitable.

8. (12 marks)

In 2020, the City of Joondalup Council hired a consultant to estimate the proportion of residents who owned a small business.

(a) The consultant decides to estimate a 95% confidence interval for the proportion to within an error of 0.01. What minimum sample size should be selected?
(***NB: When the sample proportion is unknown*** $\hat{p} = 0.5$ ***should be used.***) [2]

$$n > \left(\frac{1.96\sqrt{0.5(0.5)}}{0.01} \right)^2 = 9604$$

✓ correct z
✓ n

(b) As a result of staff shortages in the Council the maximum sample size that can be managed is only 500. What is the maximum margin of error in estimating a 99% confidence interval? [2]

$$E = 2.5758\sqrt{\frac{0.5(0.5)}{500}} = 0.058$$

✓ correct z
✓ E

It is estimated that 20% of new businesses fail within the first year. The City of Joondalup now wishes to work with the Business Advisory Council (BAC). The BAC takes a random sample of 500 new businesses that started in January 2020.
By January 2021, 90 of the 500 new businesses had failed. The 95% confidence interval for the proportion of businesses that fail in the first year was (0.1463 , 0.2136).

The BAC believes that the proportion of new businesses that fail within a year can be reduced by providing financial advice. In 2021 they took another random sample of 500 businesses that started in January 2021 and provided them with regular financial advice. In this random sample, at the end of the year 80 businesses had failed.

(c) Calculate the sample proportion and its margin of error at the 95% confidence level. [2]

$$\hat{p} = \frac{80}{500} = 0.16 \qquad E = 1.96\sqrt{\frac{0.16(0.84)}{500}} = 0.0321$$

✓ $\hat{p}$
✓ E

(d) Calculate a 95% confidence interval for the proportion of businesses that failed. [2]

$$(0.1279, 0.1921)$$

✓ lower
✓ upper

(e) What do you conclude regarding the value of the financial advice provided to the new businesses? [2]

Comparing this CI with previous of (0·1463, 0·2136) there is overlap.

∴ does not appear that the financial advice has reduced the proportion of businesses that fail in the first year.

✓mention overlap

✓conclusion

(f) If the sample size was reduced, what would be the effect on the confidence interval? Justify your answer. [2]

Width of CI would increase because Margin of Error of $\hat{p}$ would increase

∴ increasing error.

✓width ↑

✓↑ MoE or ↑ error